Lagrange's interpolation formula (for unequal Intervals)

If the values of independent variable are not equally spaced, we will use Lagrange's interpolation formula.

Let $y = f(x)$ be a function such that $f(x)$ takes the values $y_0, y_1, y_2, \ldots, y_n$ corresponding to ~~the~~ $x = x_0, x_1, \ldots x_n$.

i.e., $y_i = f(x_i)$; $i = 0, 1, 2, \ldots, n$.

Now, there are $(n+1)$ paired values (x_i, y_i), $i = 0, 1, 2, \ldots, n$ and hence $f(x)$ can be represented by a polynomial function of degree ~~or~~ n in x.

We will select that $f(x)$ as follows.

$$\begin{aligned} f(x) = {} & a_0 (x - x_1)(x - x_2) \ldots (x - x_n) \\ & + a_1 (x - x_0)(x - x_1) \ldots (x - x_n) \\ & + a_2 (x - x_0)(x - x_1) \ldots (x - x_n) + \ldots \\ & + a_i (x - x_0)(x - x_1) \ldots (x - x_{i-1})(x - x_{i+1}) \ldots (x - x_n) + \ldots \\ & + a_n (x - x_0)(x - x_1) \ldots (x - x_{n-1}) \end{aligned} \quad \rightarrow (1)$$

Note: The term in which a_i occurs has the factor $(x - x_i)$ lacking.

This is true for all values of x.

Substituting in (1), $x = x_0$, $y = y_0$, we get

$$y_0 = a_0 (x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_n)$$

$$\therefore a_0 = \frac{y_0}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \ldots (x_0 - x_n)}.$$

$|||^{ly}$ $x = x_1$; $y = y_1$, we have

In the same way, we get

$$a_1 = \frac{y_1}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3) \ldots (x_1 - x_n)}$$

$$a_2 = \frac{y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)}$$

$\vdots$

$$a_n = \frac{y_n}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})}$$

Substituting these values of a's in ①, we have

$$y = f(x) = \qquad \rightarrow ②$$

Eqn ② is called Lagrange's interpolation formula for unequal intervals.

Different form of LIF:

LIF can also be written as

$$f(x) = \sum_{i=0}^{n} \frac{\Pi_n(x)}{(x-x_i)\,\Pi_n'(x_i)}\, y_i \quad \text{where}$$

$$\Pi_n(x) = (x-x_0)(x-x_1)\dots(x-x_n) \quad \text{and}$$

$$\Pi_n'(x) = \frac{d}{dx}\left[\Pi_n(x)\right].$$

Problem

1. Using LIF, find $y(10)$.

x :	5	6	9	11
y :	12	13	14	16.

x :	7.	8	9	10
y :	3	1	1	9

find $y(9.5)$.

Sol

By LIF, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdot y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{(x-x_0)(x-x_2)(x-x_3)\cdot y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ \qquad y_2 + \qquad y_3$$

Putting $x = 10$;

$$y(10) = 14.6667.$$